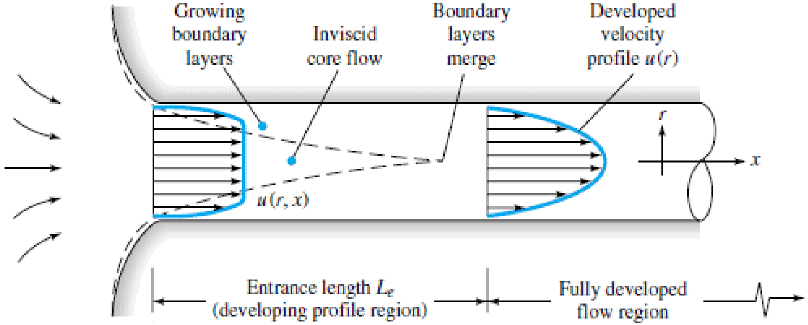
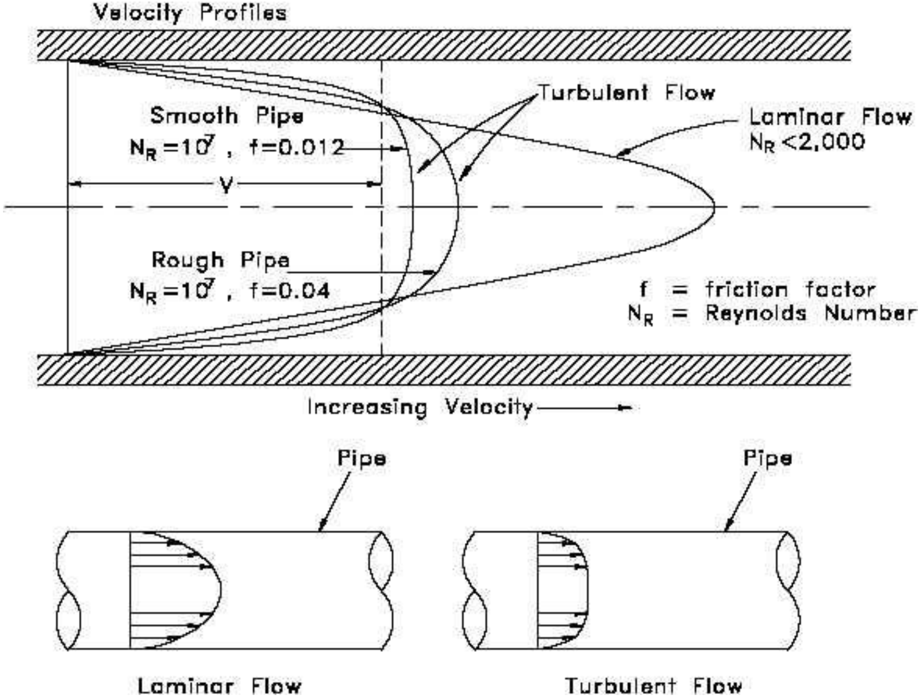
4. Boundary Layers– *Internal Flow*



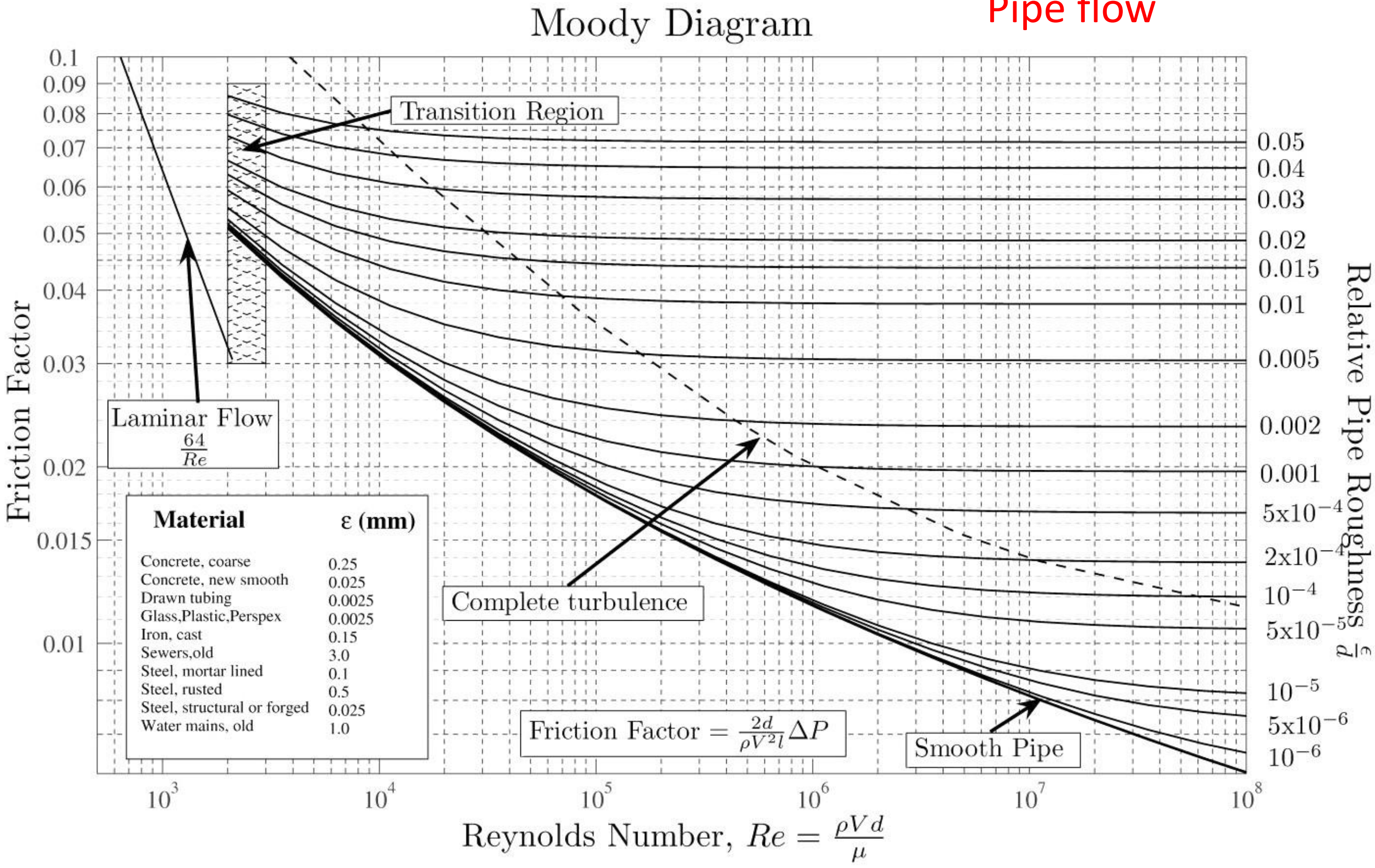
Velocity Profile of Fluid in a Pipe (laminar Flow)

$$u = u_{max}\left(1 - \frac{r^2}{R^2}\right)$$

$$\tau_w = \left|\mu \frac{du}{dr}\right|_{r=R}$$

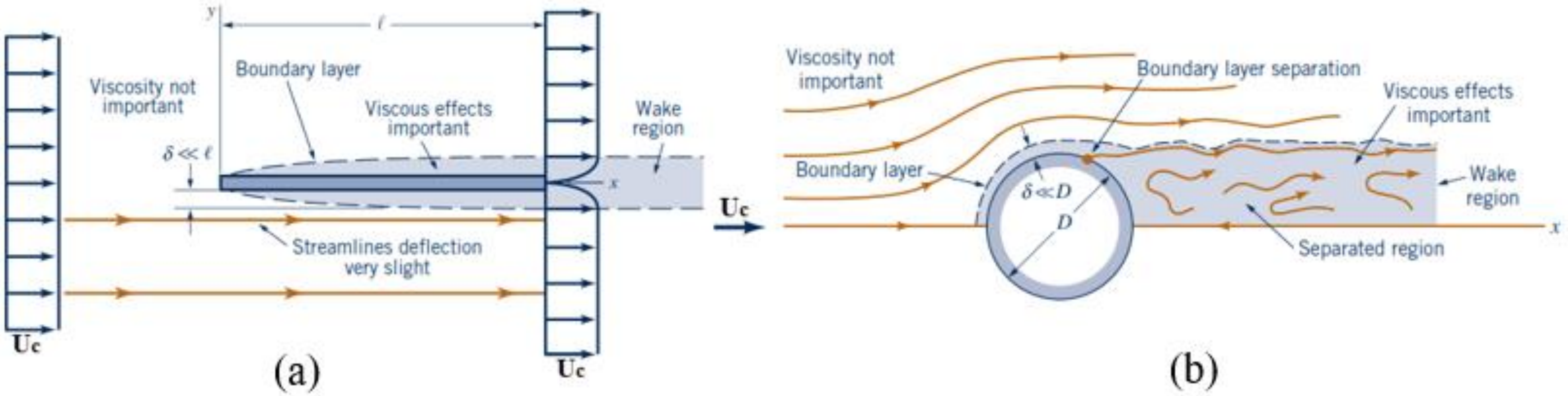


Pipe flow



Material	ε (mm)
Concrete, coarse	0.25
Concrete, new smooth	0.025
Drawn tubing	0.0025
Glass,Plastic,Perspex	0.0025
Iron, cast	0.15
Sewers,old	3.0
Steel, mortar lined	0.1
Steel, rusted	0.5
Steel, structural or forged	0.025
Water mains, old	1.0

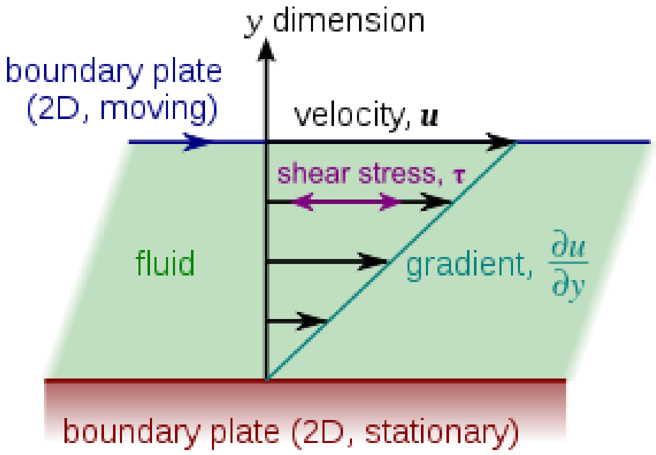
Boundary Layers– *External Flow*



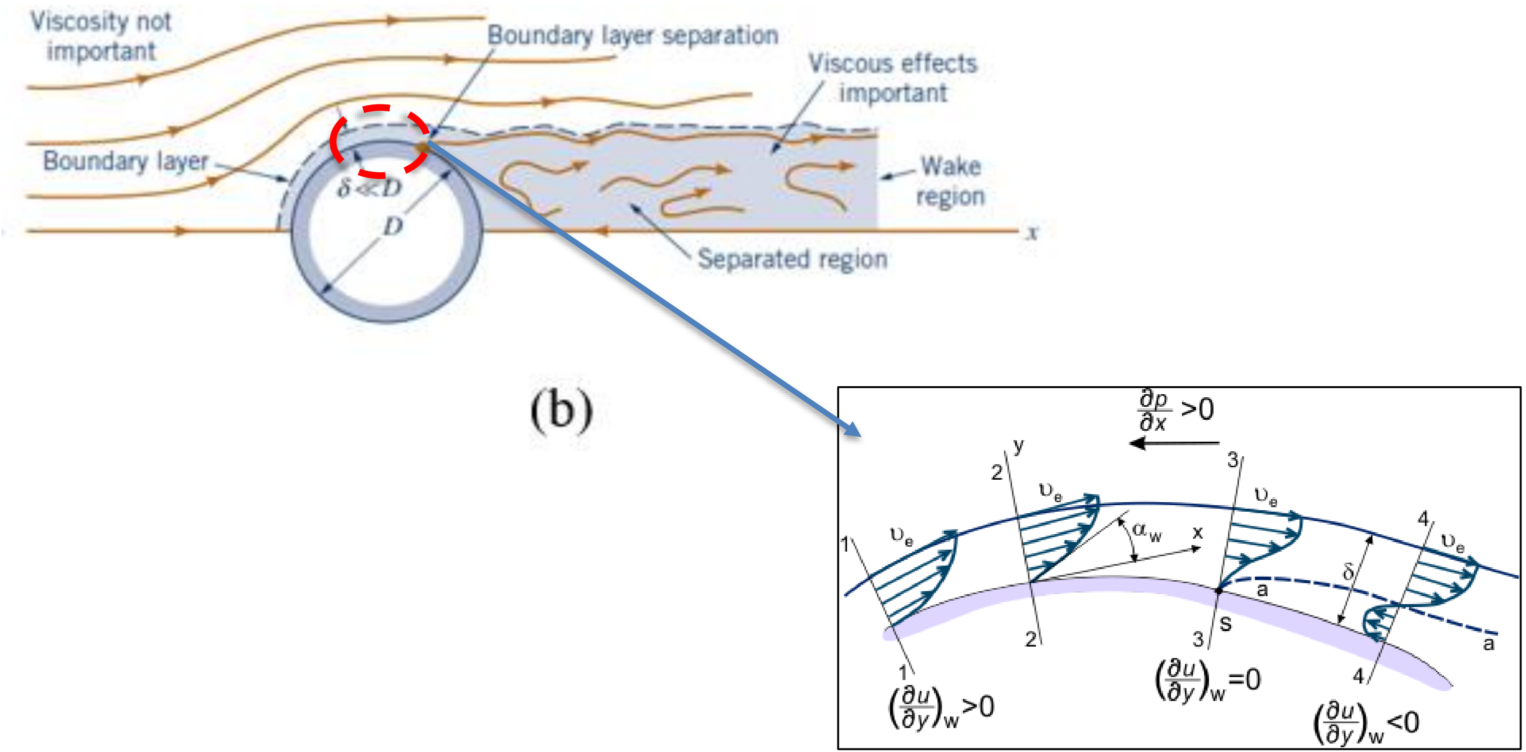
Discuss which type of forces are important here

Boundary Layers– *wall shear stress*

- τ is force per unit area
- Laminar flow: $\tau = \mu \frac{du}{dy}$



Drag with pressure gradient:



(b)

3. Boundary Layers– *Prandtl's BL equations*

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \approx U\frac{dU}{dx} + \frac{1}{\rho}\frac{\partial \tau}{\partial y}$$

$$\tau = \tau_{lam} + \tau_{turb}$$

$$\tau_{lam} = \mu\frac{\partial u}{\partial y}$$

$$\tau_{turb} = -\rho\overline{u'v'} = \mu_t\frac{\partial \bar{u}}{\partial y}$$

Assumptions:

$$v \ll u$$

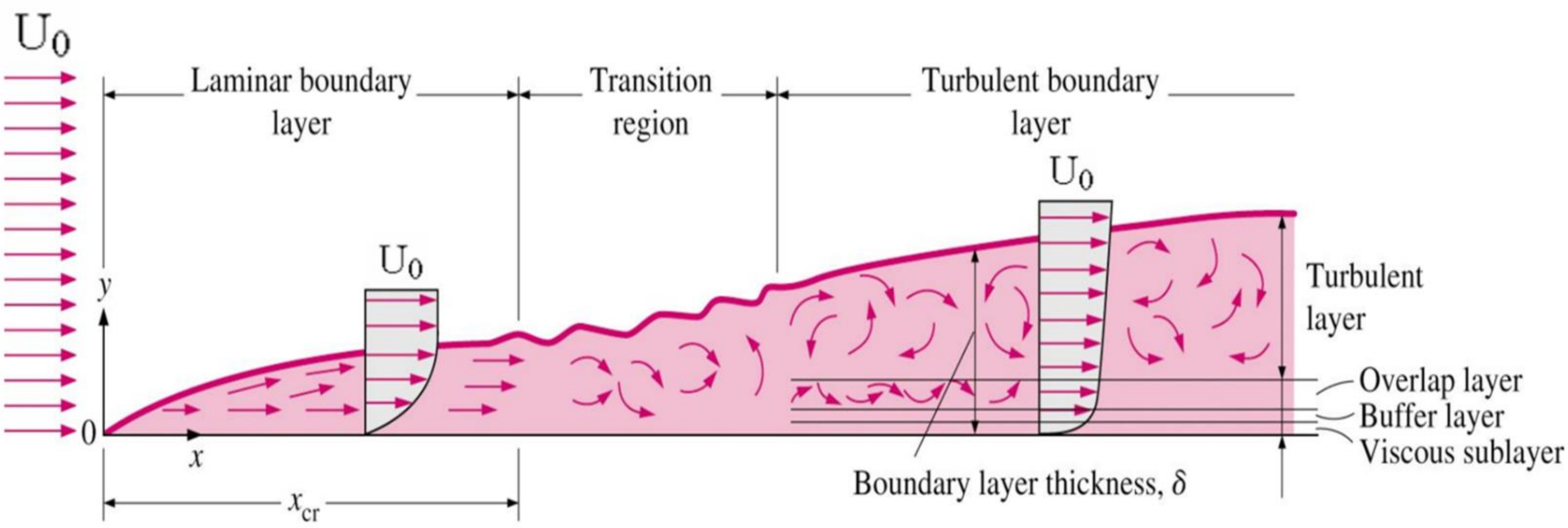
$$\frac{\partial u}{\partial x} \ll \frac{\partial u}{\partial y}$$

$$Re_x = \frac{Ux}{\nu} \gg 1$$

$$p \approx p(x)$$

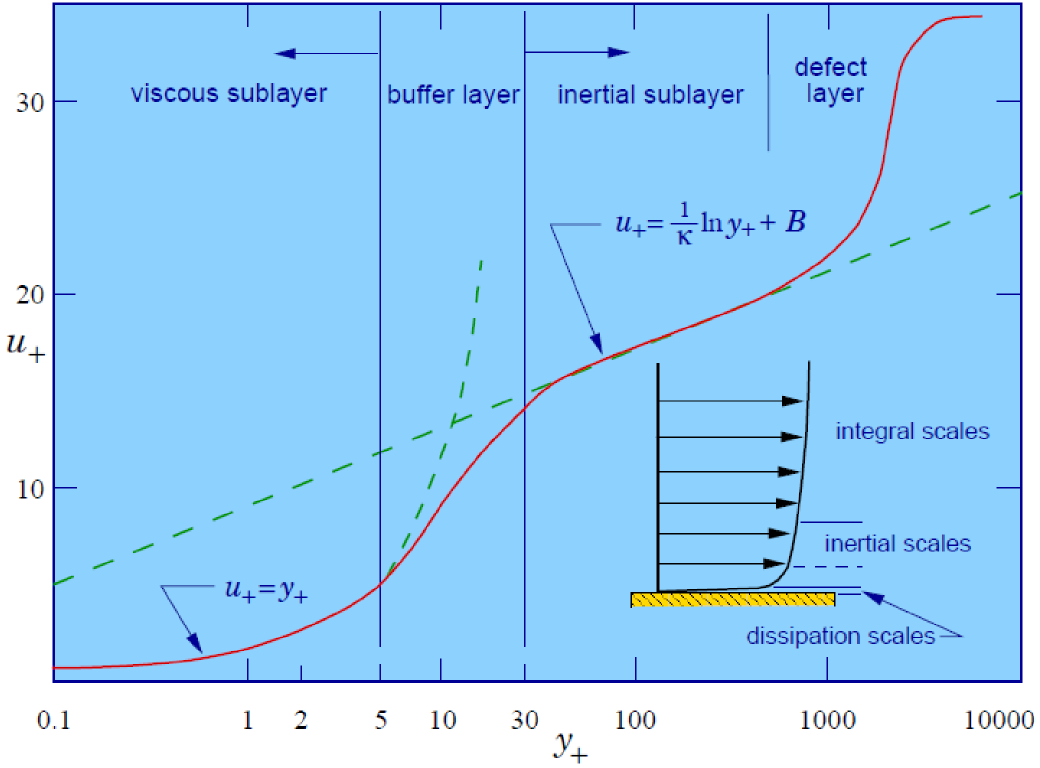
$$\frac{dp}{dx} = -\rho U\frac{dU}{dx}$$

Boundary Layers– *Turbulent Flow close to Wall*



(Reference: Y. Cengel, Fluid Mechanics: Fundamentals and Applications)

Turbulent Boundary Layer



Turbulent BL consist of 3 distinct regions :

1. *<u>Wall layer</u>: Viscous shear dominates*
2. *<u>Outer layer</u>: Turbulent shear dominates*
3. *<u>Overlap layer</u>: Both types are important*

Turbulent Boundary Layer

In the near-wall region, the mean velocity is independent of the free-stream flow «far» from the wall.

It depends on distance from the wall y, density ρ, viscosity μ and wall shear stress τ_w

Definitions:

Friction velocity: $u_\tau = \sqrt{\frac{\tau_w}{\rho}}$ → Not a real flow velocity, but has the dimensions $[LT^{-1}]$

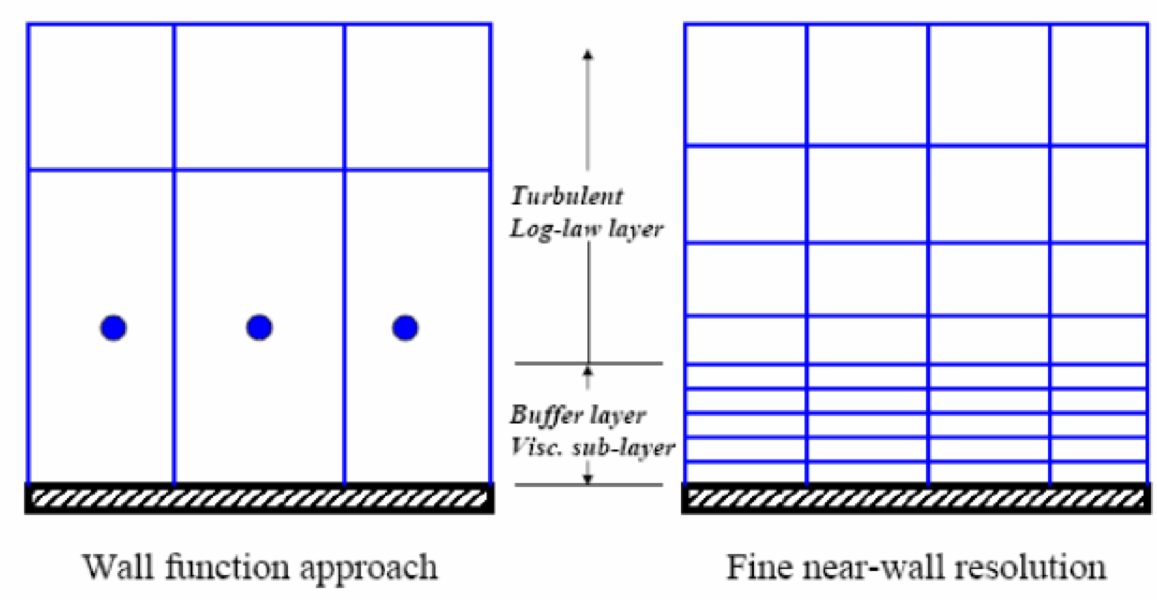
Local Re-number: $Re_y = \frac{uy}{\nu}$ → $y^+ = \frac{u_\tau y}{\nu}$

! IMPORTANT non-dimensional parameters in CFD: $u^+ = \frac{u}{u_\tau}$ and $y^+ = \frac{u_\tau y}{\nu}$

Turbulent Boundary Layer

Need to consider this when making a mesh:

1. Choose to model the whole BL: $y^+ < 1 \ (5)$
2. Use *wall functions* to bridge the region between the inner-wall layer (laminar) and the log-layer (turbulent): $y^+ > 30$



Boundary Layers – *Estimate y+*

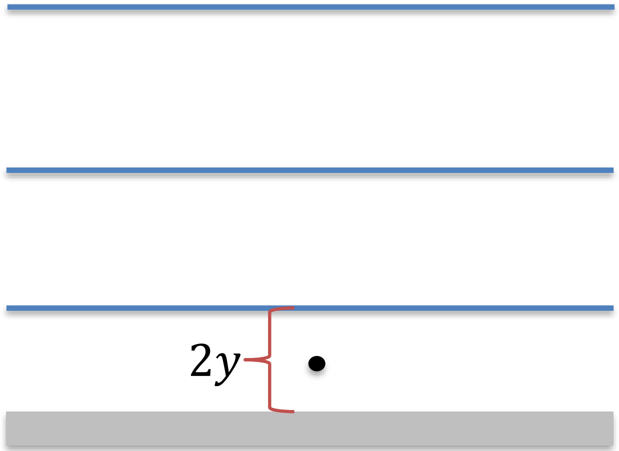
1. Compute the Re_x number (x-boundary layer length)

2. Estimate skin friction coefficient. Ex. Schlichting:

$$C_f = [2\log_{10}(Re_x) - 0.65]^{-2.3} \quad for \;\; Re_x < 10^9$$

3. Calculate wall shear stress:

$$\tau_w = C_f \cdot \frac{1}{2}\rho U_\infty^2$$



4. Friction velocity:

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}}$$

5. Wall distance:

$$y = \frac{y^+ \mu}{\rho u_\tau}$$