Statistical methods

- Mean value and standard deviations
- Standard statistical distributions
- Linear systems
- Matrix algebra

Statistical methods – Generating random numbers

- MATLAB has many built-in functions to generate random numbers
  - `unidrnd(N)` Generates random numbers from the discrete uniform distribution with maximum N
  - `normrnd(mu,sigma)` Generates random numbers from the normal distribution with mean parameters mu and standard deviation sigma
- MATLAB has many built-in functions to find statistical parameters from data set

`sum`	sum	`min`	minimum
`mean`	average	`max`	maximum
`var`	variance	`median`	median
`std`	standard deviation	`iqr`	quartile difference
`cov`	covariance	`range`	variance width
`corr`	correlation	`prctile`	percentile

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Statistical methods – standard statistical functions

- Plotting of stochastic variables

```
x = -4:0.1:4;            % vector with x-values
y = normpdf(x);          % vector with y-values
plot(x,y,'r-');          % plot with red line
xlabel('x');             % description on x-axis
ylabel('density');       % description on y-axis
title('Standard normal distribution')
grid on                  % make a grid
box on                   % make a box around figure
```

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Statistical methods – Exercise

- Make a file with the following data

```
22    24    25    21    23    26    % Ages
178   183   194   168   172   188   % Heights
65    86    72    70    75    80    % Weights
```

- Read the file into MATLAB
  - Calculate mean value of age, height and weight
  - Calculate BMI for all persons
  - Calculate mean value of BMI

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Linear systems – Matrix algebra

- Linear systems – System of linear equations
  Suppose you are given the two linear equations below.
  MATLAB is a powerful tool to help you solve this set of equations.

```
x + 2y = 6
x - y = 0
```

- Matrix Algebra – Linear algebraic equations
  - A x = B
  - Examples from Matematikk 1

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Linear systems – Matrix algebra

- Using matrix algebra, we must write the set of linear equations in terms of vectors and matrices.
- Our two equations can then be written as

$$\begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} 6 \\ 0 \end{Bmatrix}$$

- Which can be written as

$$\mathbf{A}\,\mathbf{x} = \mathbf{B}$$

  where **A** is the known coefficient matrix, **X** is the variable matrix and **B** is the constant matrix
- To solve for **X**, we find the inverse of **A** (provided the inverse exists) and then pre-multiply the inverse to the **B**-matrix, i.e.

$$\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$$

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Linear systems – Matrix algebra

- The MATLAB code for this operation can be written as follows

```
% To solve two simultaneous linear equations
  A = [1 2; 1 -1];
  B = [6;0];
  X = inv(A)*B
```

- Alternatively we can simply use the left division operator. Gaussian elimination is used when performing this operation.

```
  % Solving linear equations by matrix division
  A = [1 2; 1 -1];
  B = [6;0];
  X = A\B
```

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Linear systems – Matrix algebra

- Given the following set of linear equations

$$5x = 3y - 2z + 10$$
$$8y + 4z = 3x + 20$$
$$2x + 4y - 9z = 9$$

- This set can be rearranged to

$$5x - 3y + 2z = 10$$
$$-3x + 8y + 4z = 20$$
$$2x + 4y - 9z = 9$$

- Which now has the form **AX = B,** where **A**, **X** and **B** are given as

$$A = \begin{bmatrix} 5 & -3 & 2 \\ -3 & 8 & 4 \\ 2 & 4 & -9 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 10 \\ 20 \\ 9 \end{bmatrix}$$

Coefficient matrix | Vector of unknowns | Vector of constants

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Linear systems – Matrix algebra

- The MATLAB code to solve this set of equations can be written as

```
>> A = [5 -3 2; -3 8 4; 2 4 -9];
>> B = [10; 20; 9];
>> X = A\B
X =
        3.4442
        3.1982
        1.1868
```

- To check the result, we can write

```
>> C = A*X
C =
        10.0000
        20.0000
         9.0000
```

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Linear systems – Exercise

1. Create the variables to include the following matrices

$$\texttt{A} = [12\ 17\ 3\ 4] \qquad \texttt{B} = \begin{bmatrix} 5 & 8 & 3 \\ 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix} \qquad \texttt{C} = \begin{bmatrix} 22 \\ 17 \\ 4 \end{bmatrix}$$

   a) Assign the second column of `A` to the variable `x1`.
   b) Assign the third column of `B` to the variable `x2`.
   c) Assign the third row of `B` to the variable `x3`.
   d) Assign the first three variables of `A` as the first row in `x4`, and all the variables in `B` as the second, third and fourth rows.

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Linear systems – Exercise

2. If matrix A is defined using the MATLAB code `A = [1 3 2; 2 1 1; 3 2 3]`, which command will produce the following matrix

$$B = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$$

3. Create variables to represent the following matrices

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \\ -1 & 2 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 1 & 2 \end{bmatrix}$$

   a) Try performing the following operations: `A+B`, `A*B`, `A+C`, `B*A`, `B-A`, `A*C`, `C-B`, `C*A`. What are the results? What error messages are generated? Why?

   b) What is the difference between `A*B` and `A.*B`?

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Linear systems – Exercise

4. Solve the following systems of linear equations. Remember to verify your solution

   a)
$$\begin{aligned} -2x + y &= 3 \\ x + y &= 10 \end{aligned}$$

   b)
$$\begin{aligned} 5x + 3y - z &= 10 \\ 3x + 2y + z &= 4 \\ 4x - y + 3z &= 12 \end{aligned}$$

   c)
$$\begin{aligned} x_1 - 2x_2 - x_3 + 3x_4 &= 10 \\ 2x_1 + 3x_2 + x_4 &= 8 \\ x_1 - 4x_3 - 2x_4 &= 3 \\ -x_2 + 3x_3 + x_4 &= -7 \end{aligned}$$

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Linear systems – Exercise

5. Solve the following systems of linear equations. Remember to verify your solution

a)
```
-a + 4b - 6c = pi
8a + b = 2
a cos(pi/6) - exp(2.3) c = 0
```

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Linear systems – Exercises from Matematikk 1

1.1 Solve each of the following systems

1. $x_1 + 7x_2 = 4$
   $-2x_1 - 9x_2 = 2$

3. $x_1 - 3x_2 = 4$
   $-3x_1 + 9x_2 = 8$

13. $x_2 + 5x_3 = -4$
    $x_1 + 4x_2 + 3x_3 = -2$
    $2x_1 + 7x_2 + x_3 = -1$

Determine if the following system is consistent

17. $-2x_1 - 3x_2 + 4x_3 = 5$
    $x_2 - 2x_3 = 4$
    $x_1 + 3x_2 - x_3 = 2$

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Linear systems – Exercises from Matematikk 1

1.2 Find the reduced row echelon form of the following matrices
Hint: Use the MATLAB command `rref`
(try `help rref` to see how the command works)

5. a $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 6 & 7 & 8 & 7 \end{bmatrix}$

5. b $\begin{bmatrix} 1 & 3 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 1 \end{bmatrix}$

Find the general solution of the systems whose augmented matrices are given as

7. $\begin{bmatrix} 1 & 0 & 2 & 5 \\ 2 & 0 & 3 & 6 \end{bmatrix}$

9. $\begin{bmatrix} 0 & 3 & 6 & 9 \\ -1 & 1 & -2 & -1 \end{bmatrix}$

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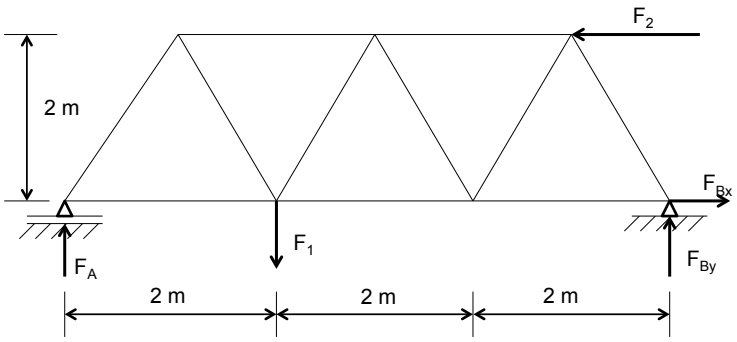
Linear systems – Exercises from Statikk

Find the reaction forces F_A and F_B

$F_1 = -15$ kN
$F_2 = -5$ kN



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